The Effect of Variation in Atmospheric Absorption
on Millimetric, Terrestrial, Telecommunications Links

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Abstract

When planning the link budget for millimetre and sub-millimetre wave telecommunications links, the attenuation by atmospheric gasses is taken into account. However, at millimetric frequencies above 50 GHz, it may become necessary to account for the temporal variations in absorption when calculating fade margins. Attenuation due to atmospheric absorption varies with temperature, barometric pressure and humidity, and the amplitude of variation can become significant compared to rain fade margins. This paper examines the variation in absorption between 10 and 1000 GHz derived from meteorological time-series. This variation is compared to the attenuation time-series experienced by two experimental, 5 km links operating at 54.5 and 56.5 GHz in the southern UK. A method is developed to predict the average annual distribution of specific attenuation due to atmospheric absorption, based on easily obtainable meteorological parameters. A further method is developed to integrate this variation into existing models of annual fade distribution and these results are compared to measured distributions.
1. Introduction

The International Telecommunication Union (ITU) has produced a range of recommendations that allow the prediction of the link budget and the distribution of fades experienced by terrestrial line-of-sight links in an average year. ITU-R Rec. P.530-10 [2001] includes models of both clear-air and rain fade. The attenuation due to absorption by atmospheric gasses is explicitly listed as an effect to be included in the link budget but variation in this attenuation is not included in the prediction model of fade distributions. Rec. P.530-10 does state: “On long paths at frequencies above about 20 GHz, it may be desirable to take into account known statistics of water vapour density and temperature in the vicinity of the path. Information on water vapour density is given in Recommendation ITU-R P.836.” This statement suggests that temporal variation of absorption fade may be significant at these higher frequencies, on long paths, but no procedure for incorporating this effect is provided.

At increasing millimetre and sub-millimetre wavelengths, from 10 GHz to 1 THz, the mean specific attenuation due to absorption by atmospheric gasses exhibits a general increase punctuated by very highly attenuated frequencies corresponding to the absorption lines of oxygen and water vapour. Between 1 THz and 10 THz these absorption lines are so close together in frequency that the atmosphere at lower altitudes is essentially opaque.

Above 55 GHz, absorption by atmospheric gases is an important constraint on the use of telecommunications bands due to the high transmit power required for even modest path lengths. However, other aspects of these bands are attractive such as the bandwidth available, the small antennae sizes and the short reuse distances. The margin required to counter rain attenuation determines the availability of these systems. Specific attenuation due
to rain scatter and absorption increases monotonically between 10 GHz and 100 GHz, where it plateaus and then slowly declines. In this paper it is postulated that at specific frequency ranges above 50 GHz, and at all frequencies above 350 GHz, temporal variation in atmospheric absorption is more significant than rain fading. Higher frequency millimetric and sub-millimetric links are likely to be short and with low fade margins due to hardware constraints of radiated power, see Gibbins and Norbury [2002]. In this paper we show that low fade margin systems in particular will be adversely effected by variation at atmospheric absorption.

This paper investigates the fading experienced by two experimental links operating at 54.5 and 56.5 GHz, located in the southern UK. The links operate along the same 5 km path between Sparsholt (Lat. 51° 5’ 2”, Long. 1° 23’ 27”) and South Wonston in Hampshire, UK. These frequencies are close to a group of oxygen absorption lines around 60 GHz and experience mean specific attenuations due to atmospheric absorption of 3.1 and 8.7 dB/km respectively. Along the 5 km path, these attenuations of 15.5 and 43.5 dB were included in the link budget when the experiment was designed. In Section 2 the measured, clear-air, fade time-series are compared with atmospheric absorption time series derived from meteorological measurements of pressure, dew point and temperature. In Section 3 the specific attenuation is shown to be Normally distributed and an expression for the average annual distribution is presented. In Section 4 a method is developed for predicting the variance of specific attenuation in different climates. Finally, in Section 5, a method is suggested for incorporating fading due to temporal changes in atmospheric absorption into existing models of average annual fade distributions.

2. Atmospheric Absorption on the Experimental Links
Molecules present in the atmosphere, principally water vapour and oxygen, absorb millimetre and sub-millimetre waves. The amount of attenuation caused by this absorption process depends upon the temperature and partial pressure of these gases. ITU-R Rec. P.676-5 [2001] provides a method of calculating the specific attenuation caused by the atmosphere given meteorological information. At 38 GHz the absorption is approximately 0.15 dB/km at sea level. However, 54.5 GHz and 56.5 GHz are close to a group of oxygen absorption lines around 60 GHz where specific attenuation exceeds 10 dB/km at sea level. Temporal variations in temperature and humidity leads to receive power level changes on the 5 km links at 54.5 and 56.5 GHz.

The meteorological station at Sparsholt provides measurements of air temperature $T_C$ °C, dew point temperature $T_d$ °C and barometric pressure $P$ hPa; every ten seconds. The inputs to Rec. ITU-R P.676-5 are the dry air pressure $p$ hPa, the water vapour partial pressure $e$ hPa and the temperature $T$ °K. The inputs are calculated from the available data using the following relationships:

$$e = 6.1078 \times 10^{\frac{7.5p_e}{T_d+237.3}} \text{ hPa}$$ (1)

$$p = P - e \text{ hPa}$$ (2)

$$T = T_C + 273.15 \text{ °K}$$ (3)

Figure 1 illustrates a week of measured receive power on the 54.5 GHz and 56.5 GHz links, from 16 to 22 June 2004. Also shown is the variation due to changes in atmospheric absorption, predicted using Rec. ITU-R P.676-5 and measured meteorological parameters. The mean level has been chosen arbitrarily to allow easy comparison between the measured
and predicted series. No rain occurred during this period, which was characterised by hot days and cool nights. The variation in the measured receive power on the 56.5 GHz link closely follows the prediction, with a circadian 5 dB oscillation. Other clear-air effects, such as multi-path and scintillation, cause faster variation that appears as additive noise on top of the larger variation due to atmospheric absorption. The 54.5 GHz measured receive power time series shares some resemblance with the predicted time series but the other clear-air effects lead to higher amplitude variation, obscuring the absorption effects.

Figure 1: Receive power time-series for the 5 km, 54.5 GHz and 56.5 GHz links for the period 16/6/2004 - 22/6/2004. These are compared with the temporal variation expected due to changes in atmospheric absorption (shifted to an arbitrary mean level)

3. Average Annual Distribution Models

The specific attenuation at 54.5 and 56.5 GHz has been calculated from one-minute sampled meteorological parameters for the two-year period 1/10/2002 until 30/9/2004. Figures 2 A&B illustrate the cumulative distributions of specific attenuation as Normal distribution plots. Samples from a Normal distribution appear on a straight line on these plots. The calculated temporal distribution of specific attenuation is well modelled as a Normal
distribution for five standard deviations either side of the mean i.e. approximately 99.9999% of the time. This is close to the limit imposed by the finite quantity of data and the poor statistical sampling of extreme measurements.

For a mean global reference atmosphere, $e = 9.97$ hPa, $p = 1003.03$ hPa, $T = 288.15$ K, Rec. ITU-R P.835-3 [1999], the specific attenuations at 54.4 and 56.5 GHz are 3.04 and 8.49 dB/km respectively. The distributions derived from meteorological measurements have means slightly higher than those implied by a reference atmosphere. The use of the reference atmosphere leads to small errors in the link budget of approximately 0.2 and 1.0 dB for the 54.5 and 56.5 GHz links respectively.

More significant is the variance of these distributions, implying large and slow variation of receive power around the mean value, in both clear-air and rainy conditions. For the 56.5 GHz link the standard deviation is approximately 0.5 dB/km. These variations are not included in the Rec. ITU-R P.530-10 fade distribution model and become large at low time percentages.

Figure 2A

Figure 2B
Figure 2: Normal distribution plots of specific attenuation due to atmospheric absorption derived from meteorological measurements for A:54.5 GHz and B:56.5 GHz.

If the marginal distribution of specific attenuation due to atmospheric absorption $\gamma$ is well modelled as a Normal distribution with variance $\sigma_\gamma^2$, the fade relative to the mean exceeded $p\%$ of the time due to variation of atmospheric absorption is:

$$A_{\text{sat}}(\gamma) = L\sigma_\gamma \text{erfc}^{-1}\left(\frac{p}{100}\right)$$

(4)

where $\text{erfc}^{-1}$ is the inverse of the complimentary error function (normalised to be consistent with a Standard Normal distribution). This assumes that the specific attenuation is near constant over scales of the order of the link length $L$. For almost all links this assumption will be valid. For a small number of high elevation links, or links passing through extreme micro-climates, the link will need to be considered in sections. It is likely that the meteorological parameters will vary in-phase along the climatically separate parts of the link, but possibly with different amplitudes. If a link passes through two micro-climates with a length $L_1$ in a climate with variance $\sigma_1^2$ and a length $L_2$ in a climate with variance $\sigma_2^2$, then the absorption attenuation is Gaussian distributed and (4) holds with the factor $L\sigma_\gamma$ replaced by $L_1\sigma_1 + L_2\sigma_2$.

4. Predicting the Variance of Specific Attenuation

The problem remains to estimate the variance of specific attenuation due to atmospheric absorption, $\sigma_\gamma^2$, for a range of frequencies using widely available data parameters. The mean
specific attenuation $\bar{\gamma}$ varies by four orders of magnitude over the frequency range 10 to 1000 GHz and the standard deviation could be expected to vary similarly. Figure 3 compares the mean and standard deviation of specific attenuation for the frequency range 10 to 1000 GHz, calculated from the two-year time series of meteorological measurements discussed in Section 3. This figure shows that the standard deviation similarly varies by four orders of magnitude across this frequency range. Over most of this frequency range the simple linear relation $\sigma_\gamma \approx \frac{\bar{\gamma}}{4}$ holds. However, for several important frequency ranges this relationship is inadequate i.e. 10 – 13 GHz, 44 – 72 GHz and 118-121 GHz. The second and third of these ranges are associated with absorption lines.

Figure 3: Scatter plot of the mean and standard deviation of the specific attenuation due to absorption estimated from the measured meteorological time series for the interval 1/10/2002 to 30/9/2004 (+) and derived from (6) (*). The grey * indicate frequencies in the range 10-13, 43-72 and 118-121 GHz.
A model predicting $\sigma^2_\gamma$ will be derived from a power-series expansion of the relationship between specific attenuation and the meteorological parameters. This yields an expression for $\sigma^2_\gamma$ depending upon the summarising statistics of temperature, pressure etc. and the derivatives of specific attenuation. Defining the meteorological parameter vector $\mathbf{x} = \{t,e,T\}$, then variation of specific attenuation around its mean value $\bar{\gamma} = \gamma \bar{\mathbf{x}}$ may be written as a Taylor expansion:

$$\gamma \bar{\mathbf{x}} + \Delta \mathbf{x} \bar{\gamma} + \Delta \mathbf{x}^T \nabla \bar{\gamma} + \frac{1}{2} \Delta \mathbf{x}^T \mathbf{G} \Delta \mathbf{x} \cdots$$ \hspace{1cm} (5)

where $\nabla \bar{\gamma}$ is the gradient vector and $\mathbf{G}$ is the Hessian matrix, both evaluated at the mean of the meteorological parameters. Truncating the series at the second order yields an approximate expression for the variance of specific attenuation in terms of the covariances of meteorological parameters $\sigma^2_{x_ix_j}$ and the first and second order derivatives of the specific attenuation:

$$\sigma^2_\gamma = \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma^2_{x_ix_j} \left( \bar{\gamma} \frac{\partial^2 \gamma}{\partial \mathbf{x}_i \partial \mathbf{x}_j} + \frac{\partial \gamma}{\partial \mathbf{x}_i} \frac{\partial \gamma}{\partial \mathbf{x}_j} \right)$$ \hspace{1cm} (6)

The derivatives may be evaluated numerically or by symbolic differentiation of the expressions provided in Rec. ITU-R P.676-5 while the meteorological covariances need to be calculated from measured time series. For the two years of Sparsholt data considered, the mean and covariance of the meteorological parameters are:
\[ \bar{x} = \{92.6 \text{ hPa}, 9.733 \text{ hPa}, 282.65 \text{ }^\circ \text{K} \} \]  

\[ \sigma_x^2 = \begin{pmatrix} 130.0 & -13.55 & -13.29 \\ -13.55 & 10.21 & 14.25 \\ -13.29 & 14.25 & 31.27 \end{pmatrix} \]  

where the units of covariance elements are hPa^2 or hPa^0K. At 56 GHz the derivatives of atmospheric absorption are:

\[ \nabla \bar{\gamma} = \{690.180, -3.90 \bar{x} \times 10^{-2} \} \]  

\[ \bar{G} = \begin{pmatrix} -0.00015 & 1.02 & -4.465 \\ 1.02 & 78.8 & -40.26 \\ -4.465 & -40.26 & 37.5 \end{pmatrix} \times 10^{-5} \]  

Figure 3 illustrates the mean and standard deviation of specific attenuation derived from the simplified method, (6), using the meteorological parameters (7 and 8) and numerical derivatives of which (9 and 10) are examples. The method of (6) reproduces the major features present in the measured data.

Figure 4 compares the standard deviation of specific attenuation derived from the meteorological time-series with those predicted by (6). Due to truncation of the Taylor series, (5), the model under predicts the measured standard deviation by approximately 7% at \( \sigma_y = 1 \text{ dB/km} \), with the error increasing for higher standard deviations. An empirical correction may be applied based on the power law fit to figure 4: \( \hat{\sigma}_y = 1.050 \sigma_y^{1.023} \), where \( \hat{\sigma}_y \) will be a more accurate estimate of the standard deviation. This method works well except in the range 55 - 65 GHz where (6) over-estimates the variance by as much as 50%.
Using higher order terms in the Taylor series, (5), may resolve both these inaccuracies. However the resulting equations would be significantly more complex and require a large number of derivatives and statistical moments of meteorological parameters. An empirical correction over this range is likely to be more practical.

Figure 4: Scatter plot of standard deviation of the specific attenuation due to absorption estimated from the measured meteorological time series for the interval 1/10/2002 to 30/9/2004 and derived from (6) for the frequency range 10 – 1000 GHz. The dashed line indicates equality while the dotted grey line indicates the best-fit power law.

Figure 5 compares the 0.01% exceeded specific attenuation due to rain with that due to variation in atmospheric absorption, calculated using (4 and 6). The rain rate used was 22 mm/hr, the 0.01% exceeded rain rate yielded by Rec. ITU-R P.837-2 [2003] for Sparsholt. The fade due to variation of atmospheric absorption is an order of magnitude lower than that of rain around the oxygen lines at 60 GHz, but exceeds rain attenuation around the water line at 183 GHz and is consistently higher from 350 GHz to higher frequencies.
Figure 5: The 0.01% exceeded specific attenuation due to rain (dashed) and variation in atmospheric absorption (solid).

5. Extending Fade Distribution Model

An internationally recognised model for the average annual fade distribution experienced by terrestrial line-of-sight links is provided in Rec. ITU-R P.530-10 [2003]. Separate models exist for raining and non-raining (clear-air) conditions, each providing attenuation distributions, $A_R$ and $A_{CA}$, as a function of exceedance probability $p = P/100$, where $P$ is the percentage of an average year. The clear-air model also describes enhancements due to multi-path and ducting. Gibbins and Walden [2003] have proposed a variation to the rain fade component that yields an improved fit to the ITU-T database of link measurements, particularly at frequencies above 40 GHz. Rec. ITU-R P.530-10 uses a very simple approximation to combine these distributions to yield the total attenuation exceeded in an average year with probability $p$, $A_{CAR}$:

$$A_{CAR} = A_{CA} + A_R$$

[11]
The rigorously correct mathematical procedure for combining the effects of different attenuation mechanisms uses the inverse of these functions; $F_R$ and $F_{CA}$, the probability that a given attenuation is exceeded due to rain or clear-air mechanisms respectively, and the associated probability density functions: $f_R$ and $f_{CA}$. The probability that a given attenuation is exceeded in an average year is:

$$F_{CAR} \left( P_{R=0} \hat{F}_{CA} \right) = P_{R=0} \hat{F}_{R}$$

[12]

The second term in the integral is the Bayesian conditional probability that the clear-air attenuation is greater than $A-a$ given that the attenuation due to rain is equal to $a$. It is often assumed that clear-air attenuation mechanisms, including ducting and multi-path, do not occur at the same time as rain. In this case the integral (12), reduces to:

$$F_{CAR} \left( P_{R=0} \hat{F}_{CA} \right) = P_{R=0} \hat{F}_{R}$$

[13]

Where $P_{R=0}$ is the probability of rain and $\hat{F}_{CA}$ and $\hat{F}_{R}$ are the exceedance distributions normalised over periods of no-rain and rain respectively. The expression used in Rec. ITU-R P.530-10, (11), only approximates the inverse of this CDF when either clear-air or rain fade dominates the attenuation i.e. at very high or very low time percentages.

Attenuation due to variation in atmospheric absorption cannot be assumed to be mutually exclusive with other fading mechanisms. The cool, humid conditions in which atmospheric attenuation is maximised could be correlated with the onset of frontal rain. Until more information is available, we will assume that attenuation due to atmospheric absorption is
independent of the other fade mechanisms. The assumption of independence allows the simplification of (12) as a conditional probability is no longer needed. The probability of attenuation $A$ being exceeded due to a combination of atmospheric absorption and other mechanisms can then be written:

$$F_{\text{CARAA}} = \int_{-\infty}^{\infty} f_{AA} (\alpha - a) F_{\text{CAR}} (\alpha) \, da$$ [14]

where the function $f_{AA} (\alpha - a) \sim \frac{\sqrt{\pi}}{4L\sigma_{\gamma}} \exp \left( -\frac{1}{2} \left( \frac{A - a}{L\sigma_{\gamma}} \right)^2 \right)$ by manipulation of (4). Figures 6A and 6B illustrate the average annual fade exceedance predicted by the Gibbins-Walden model for the 54.5 and 56.5 GHz, 5 km links operated at Sparsholt, UK. These are compared with the prediction including the effects of variation of atmospheric absorption using (14). This increases the predicted fade at 5% of time by approximately 1 dB at 54.5 GHz and 2.5 dB at 56.5 GHz. At time percentages below 1% the change is negligible due to the low probability of extreme atmospheric absorption occurring simultaneously with deep fade due to other mechanisms. The assumption of independence strongly affects the shape of this part of the curve. Also plotted in these figures are the fade exceedance distributions for the two links, measured over the two-year period 1/10/2002 to 1/10/2004. At time percentages below 0.05% the limited measurement range of the receiver affects the measured incidence of attenuations. This leads to the plateau present in the measured exceedances around 35 dB. In both cases the measured data is a better fit to the predicted curve including variation of atmospheric absorption, particularly in the range 50% to 1% of time.
Figure 6A

Figure 6A&B: Fade exceedance distribution for the 5 km, A: 54.5 GHz and B: 56.5 GHz, Sparsholt links measured over the interval 1/10/2002 to 30/9/2004 (dotted) compared to the prediction of the Gibbins-Walden model (solid) and the Gibbins-Walden model with correction for variation in atmospheric absorption (dashed).

The measured attenuation exceedance curves can be moved up or down by the choice of reference receive power level. The reference level used was the long term average of the daily median receive power, excluding periods of rain. The algorithm is specified in the Ofcom report on this experiment, Paulson [2004]. The error in this process is estimated to be 0.5 dB at most.

6. Conclusions

Fading due to temporal variation in absorption by atmospheric gases has been shown to be an important mechanism for two millimetric frequency ranges near absorption lines and for all frequencies above 350 GHz. This mechanism is not currently considered in recognises models of fade distributions. The distribution of specific attenuation due to atmospheric
absorption has been shown to be Normally distributed. Methods have been developed for estimating its variance and incorporating the effect into annual fade distributions.
Notation

\( T_C \) \quad \text{air temperature in } ^\circ \text{C}

\( T_d \) \quad \text{dew point temperature } ^\circ \text{C}

\( P \) \quad \text{barometric pressure hPa}

\( p \) \quad \text{dry air pressure hPa}

\( e \) \quad \text{water vapour partial pressure hPa}

\( T \) \quad \text{temperature } ^\circ \text{K}

\( \gamma \) \quad \text{specific attenuation due to atmospheric absorption } \text{dB/km}

\( \sigma^2_\gamma \) \quad \text{variance of specific attenuation due to atmospheric absorption } (\text{dB/km})^2

\( L \) \quad \text{link length}

\( \nabla \gamma \) \quad \text{gradient vector of } \gamma \text{ evaluated at the mean of the meteorological parameters}

\( \overline{G} \) \quad \text{Hessian matrix of } \gamma \text{ evaluated at the mean of the meteorological parameters}

\( \sigma^2_{x,x_j} \) \quad \text{covariances of meteorological parameters}

\( A_R \) \quad \text{link attenuation due to rain as a function of exceedance probability}

\( A_{CA} \) \quad \text{link attenuation due to clear-air effects as a function of exceedance probability}

\( A_{CAR} \) \quad \text{link attenuation due to rain and clear-air effects}

\( f_R, F_R \) \quad \text{pdf and complimentary cumulative distribution of rain attenuation}

\( f_{CA}, F_{CA} \) \quad \text{pdf and complimentary cumulative distribution of clear-air attenuation}

\( f_{AA} \) \quad \text{pdf of atmospheric absorption attenuation}

\( F_{CARAA} \) \quad \text{complimentary cumulative distribution of clear-air, rain and absorption attenuation}
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References


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**Figure Captions**

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Figures

Figure 1

Figure 2A

Figure 2B