Subdivision Surface Fitting to A Dense Mesh Using Ridges and Umbilics

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Subdivision Surface Fitting

Dense triangle mesh

Coarse control mesh

Subdivision surface
Outline

• Choosing features for control mesh
• Constructing connectivity
• Hausdorff distance including curvature vectors
• Computing the control mesh
• Examples and Conclusions
Existing Methods

• Mesh simplification
• Segmentation
• Global parameterisation
• Curvature lines
Our Method

• The natural ridge-joined connectivity of umbilics and ridge-crossings is used as the connectivity of control mesh for subdivision.
• Preserving and aligning with the salient features
• Curvature sensitive distance metric for automatic construction of connectivity
Features on Surfaces

Ridges on implicit polynomial surfaces (a) Soap shape; (b) Rounded cube; (c) Rounded octahedron.
Feature Extraction

**Monge form**: A local surface can be expressed as

\[
z = \frac{1}{2} \left( k_1 x^2 + k_2 y^2 \right) + \frac{1}{6} \left( b_0 x^3 + 3 b_1 x^2 y + 3 b_2 xy^2 + b_3 y^3 \right) \\
+ \frac{1}{24} \left( c_0 x^4 + 4 c_1 x^3 y + 6 c_2 x^2 y^2 + 4 c_3 xy^3 + c_4 y^4 \right) + O(x, y)^5
\]

- and \( y \) -axes are principal directions
Ridges, Umbilics and Ridge-crossings

• Ridges are points of extrema of principal curvature along their curvature lines.
  
  \[ b_0 = \frac{dk_1}{dx} \text{ vanishes, i.e. } b_0 = 0; \]
  
  or \[ b_3 = \frac{dk_2}{dy} \text{ vanishes, i.e. } b_3 = 0. \]

• Umbilics are points where the two principal curvatures have the same value
  
  \[ k_1 = k_2 \]

• The ridge-crossings are intersections of two ridges
  
  \[ b_0 = b_3 = 0 \]
Classification of Umbilics by Ridge Configuration

(a) Hyperbolic umbilic  (b) Symmetric elliptic umbilic  (c) Un-symmetric elliptic umbilic
Example of Extracted Features

Extracted ridges, umbililcs and ridge-crossings from a Catmull-Clark subdivision surface of a car model.
Why Ridges and Umbilics?

• Ridges and umbilicus are geometrically and perceptually salient surface features, since ridges are points of extrema of principal curvatures along their curvature lines.

• Ridges and umbilicus are a decomposition of the curved surfaces as natural as the decomposition of the polyhedral surfaces into faces, edges and vertices (Thirion 1996).

• Their topological relationship provides a natural connectivity for control meshes.
Filtering of Features

Choosing features for control mesh
Distance Transform and Voronoi Diagram
Algorithm of Connectivity Construction using Distance Transform

**Algorithm 1**: 3D surface triangulation of feature points using distance transform.

```plaintext
input : A dense mesh with vertices V and a set of feature points P ⊂ V
output : A coarse triangulation of the feature points P

// initialisation
1 foreach v ∈ V do
   2 n[v] = NULL; // set v’s nearest feature point to NULL
   3 d[v] ← ∞; // set the distance d from v to it’s nearest feature point to infinity
4 foreach p ∈ P do
   5 n[p] ← p; // set p’s nearest feature point to itself
   6 d[p] ← 0; // set the distance d from p to it’s nearest feature point to zero
   7 push p to the priority queue Q;

// find the nearest feature point for each vertex
8 while Q is not empty do
   9 p ← Q pops out the top vertex with minimum d value;
   10 foreach v ∈ Adj[p] do // loop each adjacent vertex of p
      11     if d[p] + length(p, v) < d[v] then
        12         d[v] = d[p] + length(p, v);
        13         n[v] = n[p]; // update v’s nearest feature point to p’s nearest feature point
        14         push v to the priority queue Q;

// make coarse triangulation of the feature points
15 foreach triangle t in the dense mesh do
   16     get t’s three vertices v1, v2 and v3;
   17     if n[v1] ≠ n[v2] and n[v2] ≠ n[v3] and n[v3] ≠ n[v1] then
   18         make a coarse triangle using the feature points n[v1], n[v2] and n[v3];
```
Distance Fields

Constructing connectivity
Distance Metrics

- Mesh distance
- Sum of edge length
- Unwrap distance (~ mesh geodesic distance)
- Hausdorff distance including curvature vectors (~ surface geodesic distance)
Arc-length Estimation

\[ s \approx \frac{1}{\cos \frac{\theta}{2}} \left\| \left( q + \frac{l^2}{4} k_q \right) - \left( p + \frac{l^2}{4} k_p \right) \right\| \]

Hausdorff distance including curvature vectors
Hausdorff Distance including Principal Curvature Vectors

\[
\begin{align*}
\lambda k_{p1} &\quad \lambda k_{q1} \\
\lambda k_{p2} &\quad \lambda k_{q2} \\
p &\quad l &\quad q \\
\end{align*}
\]

\[
s \approx \frac{1}{\cos \frac{\theta}{2}} \max_{i=1,2} \min_{j=1,2} \left\| \left( q + \frac{l^2}{4} k_{qi} \right) - \left( p + \frac{l^2}{4} k_{pj} \right) \right\|
\]

Hausdorff distance including curvature vectors
Ridge Alignment

• We can observe that the arc-length is determined by the term $\Delta k_{pq}$ for a given chord length $l = \| q - p \|

\[ s \approx \frac{1}{\cos \frac{\theta}{2}} \left\| q - p + \frac{l^2}{4} \Delta k_{pq} \right\| . \quad \Delta k_{pq} = k_q - k_p \]
Ridge Alignment

• Consequently, two points on a ridge are more likely to be connected than two points on either side of the ridge if they have the same exact geodesic distance.

• The reason is that points on a ridge have lower variation of principal curvatures $(\frac{dk_1}{dx} = 0, \frac{dk_2}{dy} = 0)$, hence smaller estimated geodesic distance.
Ridge Alignment

(a) Unwrapped Euclidean distance  (b) the proposed Hausdorff distance

Connectivity of grassfire algorithms using different metrics
Computing of Control Mesh

Loop masks

Linear system

\[(1 - n_1 \beta) \mathbf{v}_1 + \cdots + \beta \mathbf{v}_p + \cdots = \mathbf{v}_1^\infty,\]
\[
\cdots + (1 - n_2 \beta) \mathbf{v}_2 + \cdots + \beta \mathbf{v}_q + \cdots = \mathbf{v}_2^\infty,
\]
\[
\vdots
\]
\[
\cdots + \beta \mathbf{v}_r + \cdots + (1 - n_m \beta) \mathbf{v}_m = \mathbf{v}_m^\infty,
\]
Examples: Subdivision Surface Fitting
Examples: Subdivision Surface Fitting
Fitting Errors

Quantitative comparison of the proposed method and competing methods.

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<th>Rocker arm</th>
<th>Stanford bunny</th>
<th>Igea</th>
<th>Fertility</th>
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<tr>
<td>No. of original vertices</td>
<td>10 044</td>
<td>35 947</td>
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<tr>
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<td>Hausdorff distance error</td>
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Ridge Preserving

(a) Original
(b) Proposed
(c) Simplification
(d) Kanai
Ridge Preserving

(a) Original
(b) Proposed
(c) Simplification
(d) Kanai

Examples and Conclusions
## Computational Efficiency

Computational complexity of the proposed fitting processes.

<table>
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<td>Computational times (s)</td>
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<td>Mesh simplification</td>
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<td>Total</td>
<td>87</td>
<td>234</td>
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Discussions

• Accurate surface fitting with feature alignment
• Accuracy can be improved using more robust feature extraction algorithm and optimal feature filtering method.
• Although the proposed method sacrifices some efficiency to preserve the accuracy and features, it is still reasonably efficient with total computation times of few minutes.
Conclusions

• Salient ridge features are well preserved.
• The connectivity of control mesh follows the natural ridge-joined connectivity of umbilics and ridge-crossings.
• Curvature sensitive Hausdorff distance metric improves feature alignment.
• Quad mesh construction could be the future work.
Acknowledgments

• The authors would like to thank Malcolm Sabin, Kai Hormann and Neil Dodgson for valuable suggestions.

• The authors are pleased to acknowledge the financial support of EPSRC grant EP/H030115/1.
Reference